Projects for master's dissertation (Last update: June 14, 2019)
Bhalchandra D. Thatte, Departamento de Matemática, UFMG

I am currently offering a few topics for the master's dissertation for 2020 (either to defend in July 2020 or in February 2021). The prerequisites for the following topics are basics of groups, rings, modules and categories. Álgebra Avançada and a little topology will be sufficient (to begin with). We may also study some combinatorics, graph theory, representation theory, Lie algebras, Hopf algebras and algebraic topology. Multiple projects are possible within each of the following themes; moreover, these themes are not disjoint.

1. Combinatorial reconstruction problems: can we uniquely (up to isomorphism) construct a combinatorial structure from the collection of the isomorphism classes of its substructures or from other objects or invariants of the structure? Some of the major open problems in this area are the vertex reconstruction conjecture of Ulam and Kelly (1942), the edge reconstruction conjecture of Harary (1963), and Stanley's vertex switching reconstruction conjecture; see [3].

2. Categorification is the process of replacing set-theoretic theorems by category-theoretic analogues. Categorification, when done successfully, replaces sets by categories, functions with functors, and equations by natural isomorphisms of functors satisfying additional properties. See [1], [12].

   (a) Categorification of the Jones polynomial of knots [10].

   (b) Categorification of the chromatic polynomial of fat graphs [11].

   (c) Categorification of the chromatic and the Tutte polynomials of graphs [6, 7, 8].

3. Combinatorial Hopf algebras. A Hopf algebra over a commutative ring **k** with 1 is a **k**-module with the maps of multiplication, unit, comultiplication, counit, and an antipode, with special interactions between these maps. The study of Hopf algebras arising in combinatorics was initiated by Joni and Rota [9]. We will study Hopf algebra structures arising in various contexts in graph theory, knot theory and combinatorics. See [5], [14].

   Of particular interest are the incidence algebras and coalgebras of partially ordered sets [15]. Given a poset P, we define a free **k**-module generated by the set $\{e_{xy} \mid x, y \in P, x \leq y\}$, and further define algebra and coalgebra structures. We will investigate combinatorial problems about posets in relation to the associated algebraic structures. In particular, we will be interested in various posets defined on sets of subgraphs of a graph.

4. Knots, braids, links, and 3-manifolds; invariants of Jones, Vassiliev and Witten; Kontsevich integral. A central question in the theory of knots and links is: how to determine if two knots or links are equivalent. Both Jones and Vassiliev arrived at their polynomial invariants via rather sophisticated mathematics (representation theory, von Neumann algebras, singularity theory, and so on). Later this work was simplified, making it possible to study both these invariants in elementary ways. We will study some of the more modern generalisations of these invariants due to Witten and Kontsevich. See [2], [13], [4].

References

[1] John C. Baez and James Dolan. Categorification. *arXiv Mathematics e-prints*, page math/9802029, Feb 1998.

[2] Dror Bar-Natan. On the Vassiliev knot invariants. *Topology*, 34(2):423–472, 1995.

[3] J. A. Bondy. A graph reconstructor's manual. In *Surveys in combinatorics, 1991 (Guildford, 1991)*, volume 166 of *London Math. Soc. Lecture Note Ser.*, pages 221–252. Cambridge Univ. Press, Cambridge, 1991.

[4] S. Chmutov, S. Duzhin, and J. Mostovoy. *Introduction to Vassiliev knot invariants*. Cambridge University Press, Cambridge, 2012.

[5] Darij Grinberg and Victor Reiner. Hopf Algebras in Combinatorics, 2018. Lecture notes published online.

[6] Laure Helme-Guizon. *A categorification for the chromatic polynomial*. ProQuest LLC, Ann Arbor, MI, 2005. Thesis (Ph.D.)–The George Washington University.

[7] Laure Helme-Guizon and Yongwu Rong. A categorification for the chromatic polynomial. *Algebr. Geom. Topol.*, 5:1365–1388, 2005.

[8] Edna F. Jasso-Hernandez and Yongwu Rong. A categorification for the Tutte polynomial. *Algebr. Geom. Topol.*, 6:2031–2049, 2006.

[9] S. A. Joni and G.-C. Rota. Coalgebras and bialgebras in combinatorics. *Studies in Applied Mathematics*, 61(2):93–139, 1979.

[10] Mikhail Khovanov. A categorification of the Jones polynomial. *Duke Math. J.*, 101(3):359–426, 2000.

[11] Martin Loebl and Iain Moffatt. The chromatic polynomial of fatgraphs and its categorification. *Adv. Math.*, 217(4):1558–1587, 2008.

[12] Volodymyr Mazorchuk. Lectures on algebraic categorification. *arXiv e-prints*, page arXiv:1011.0144, Oct 2010.

[13] V. V. Prasolov and A. B. Sossinsky. *Knots, links, braids and 3-manifolds*, volume 154 of *Translations of Mathematical Monographs*. American Mathematical Society, Providence, RI, 1997. An introduction to the new invariants in low-dimensional topology, Translated from the Russian manuscript by Sossinsky [Sosinskiĭ].

[14] Nigel Ray and William Schmitt. Combinatorial models for coalgebraic structures. *Advances in Mathematics*, 138(2):211 – 262, 1998.

[15] Eugene Spiegel and Christopher J. O'Donnell. *Incidence algebras*, volume 206 of *Monographs and Textbooks in Pure and Applied Mathematics*. Marcel Dekker, Inc., New York, 1997.