Universidade Federal de Minas Gerais
Instituto de Ciências Exatas – ICEx
Departamento de Matemática

Cálculo Diferencial e Integral I - Teste 4 (v1)

(1) $y = \text{sen}(2\pi x) + e^{x-1}$, $P = (1, 1)$.
(a) $y = 3x - 3$
(b) $y = \pi x - \pi$
(c) $y = 2\pi x - 2\pi$
(d) $y = (2\pi + 1)x - 2\pi$
(e) $y = (\pi - 1)x - \pi + 2$

Solução:

$y' = \cos(2\pi x) \cdot 2\pi + e^{x-1}$

$y'(1) = 2\pi \cos(2\pi) + e^0 = 2\pi + 1$

reta tangente: $y = (2\pi + 1)x + b$; $y(1) = 2\pi + 1 + b = 1 \;\Rightarrow\; b = -2\pi$

reta tangente: $y = (2\pi + 1)x - 2\pi$

(2) $y \cos(\pi xy) + x\sqrt{y} = 0$, $P = (1, 1)$.
(a) $y = \pi x - \pi + 1$
(b) $y = 3x - 2$
(c) $y = -\pi x + \pi + 1$
(d) $y = 2x - 1$
(e) $y = -x + 2$

Solução:

$$y' \cos(\pi xy) - y\,\text{sen}(\pi xy)\pi\,[y + xy'] + \sqrt{y} + \frac{x}{2\sqrt{y}}y' = 0$$

Em $(x, y) = (1, 1)$: $y'(1)\cos(\pi) - \text{sen}(\pi)\pi[1 + y'] + 1 + \dfrac{1}{2}y' = 0$

$\therefore \quad -y'(1) + 1 + \dfrac{y'(1)}{2} = 0$

$\therefore \quad y'(1) = 2$

reta tangente: $y = 2x + b$, $y(1) = 2 + b = 1 \quad \Rightarrow \; b = -1$

reta tangente: $y = 2x - 1$

Determine os seguintes limites:

(3) $\displaystyle\lim_{x \to 0} \frac{e^x - 1}{\operatorname{sen}(x)}$

<u>Solução</u>:

$$\lim_{x \to 0} \frac{e^x - 1}{\operatorname{sen}(x)} \to \frac{0}{0} \to \text{L'Hôpital}$$

$$\lim_{x \to 0} \frac{e^x - 1}{\operatorname{sen}(x)} = \lim_{x \to 0} \frac{e^x}{\cos(x)} = 1$$

(4) $\displaystyle\lim_{x \to \infty} \left[\ln(x) - x^2 \cos\left(\frac{1}{x}\right)\right]$

<u>Solução</u>:

$$\lim_{x \to \infty} \cos\left(\frac{1}{x}\right) = 1$$

$$\therefore \; \lim_{x \to \infty} \left[\ln(x) - x^2 \cos\left(\frac{1}{x}\right)\right] \to \infty - \infty$$

$$\lim_{x \to \infty} \left[\ln(x) - x^2 \cos\left(\frac{1}{x}\right)\right] = \lim_{x \to \infty} x^2 \left[\frac{\ln(x)}{x^2} - \cos\left(\frac{1}{x}\right)\right]$$

(5) $\displaystyle\lim_{x \to 1^+} \frac{\operatorname{sen}(x-1)}{\ln(x-1)}$

<u>Solução</u>:

$$\lim_{x \to 1^+} \frac{\operatorname{sen}(x-1)}{\ln(x-1)} \to \frac{0}{-\infty}$$

$$\therefore \; \lim_{x \to 1^+} \frac{\operatorname{sen}(x-1)}{\ln(x-1)} = 0$$

(6) $\displaystyle\lim_{x \to 1^+} (x-1)\operatorname{sen}(\ln(x-1))$

Solução:

$$\lim_{x \to 1^+} \ln(x-1) = -\infty$$

$$-1 \leq \operatorname{sen}(\ln(x-1)) \leq 1$$

$$-(x-1) \leq (x-1)\operatorname{sen}(\ln(x-1)) \leq (x-1) \quad ; \quad \text{para } x > 1$$

$$\lim_{x \to 1^+} (x-1) = \lim_{x \to 1^+} -(x-1) = 0$$

$$\therefore \lim_{x \to 1^+} (x-1)\operatorname{sen}(\ln(x-1)) = 0$$