

## Cálculo Diferencial e Integral I - Teste 6 (v1) - Soluções -

Calcule as seguintes integrais:

$$(1) \int_0^2 \frac{2x+1}{x^2+x+3} dx$$

$$u = x^2 + x + 3, \quad du = (2x+1)dx$$

$$\int \frac{2x+1}{x^2+x+3} dx = \int \frac{1}{u} du = \ln|u| + c = \ln|x^2+x+3| + c$$

$$\therefore \int_0^2 \frac{2x+1}{x^2+x+3} dx = \ln|x^2+x+3| \Big|_0^2 = \ln(9) - \ln(3)$$

$$(2) \int_0^1 2x^9 \sqrt{x^{10}+1} dx$$

$$u = x^{10} + 1, \quad du = 10x^9 dx$$

$$\int 2x^9 \sqrt{x^{10}+1} dx = \frac{1}{5} \int 10x^9 \sqrt{x^{10}+1} dx = \frac{1}{5} \int \sqrt{u} du = \frac{1}{5} \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c =$$

$$= \frac{2}{15} u^{\frac{3}{2}} + c = \frac{2}{15} (x^{10}+1)^{\frac{3}{2}} + c$$

$$\therefore \int_0^1 2x^9 \sqrt{x^{10}+1} dx = \frac{2}{15} (x^{10}+1)^{\frac{3}{2}} \Big|_0^1 = \frac{2}{15} \sqrt{2^3} - \frac{2}{15} = \frac{4\sqrt{2}-2}{15}$$

$$(3) \int_1^2 \frac{\ln(x)}{x} dx$$

$$u = \ln(x), \quad du = \frac{1}{x} dx$$

$$\int \frac{\ln(x)}{x} dx = \int u du = \frac{u^2}{2} + c = \frac{1}{2} \ln^2(x) + c$$
$$\therefore \int_1^2 \frac{\ln(x)}{x} dx = \frac{1}{2} \ln^2(x) \Big|_1^2 = \frac{1}{2} (\ln^2(2) - \ln^2(1)) = \frac{1}{2} \ln^2(2)$$

$$(4) \int_0^1 \frac{e^x}{\sqrt{e^x + 1}} dx$$

$$u = e^x + 1, \quad du = e^x$$

$$\int \frac{e^x}{\sqrt{e^x + 1}} dx = \int \frac{1}{\sqrt{u}} du = 2\sqrt{u} + c = 2\sqrt{e^x + 1} + c$$

$$\therefore \int_1^2 \frac{e^x}{\sqrt{e^x + 1}} dx = 2\sqrt{e^x + 1} \Big|_0^1 = 2\sqrt{e + 1} - 2\sqrt{2}$$