Universidade Federal de Minas Gerais
Instituto de Ciências Exatas – ICEx
Departamento de Matemática

Cálculo Diferencial e Integral I - Teste 6 (v1)
- Soluções -

Calcule as seguintes integrais:

(1) $\displaystyle\int_0^1 \frac{x^2}{(x^3+1)^3}\,dx$

$u = x^3 + 1, \qquad du = 3x^2dx$

$$\int \frac{x^2}{(x^3+1)^3}\,dx = \frac{1}{3}\int \frac{3x^2}{(x^3+1)^3}\,dx = \frac{1}{3}\int \frac{1}{u^3}du = \frac{1}{3}\frac{u^{-2}}{(-2)} + c =$$

$$= \frac{-1}{6u^2} + c = \frac{-1}{6(x^3+1)^2} + c$$

$$\therefore \int_0^1 \frac{x^2}{(x^3+1)^3}\,dx = \left.\frac{-1}{6(x^3+1)^2}\right|_0^1 = \frac{-1}{24} + \frac{1}{6} = \frac{1}{8}$$

(2) $\displaystyle\int_0^1 (6x^2+2)\sqrt{x^3+x+2}\,dx$

$u = x^3 + x + 2, \qquad du = 3x^2 + 1$

$$\int (6x^2+2)\sqrt{x^3+x+2}\,dx = 2\int (3x^2+1)\sqrt{x^3+x+2}\,dx = 2\int \sqrt{u}\,du =$$

$$= 2\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{4}{3}u^{\frac{3}{2}} + c = \frac{4}{3}(x^3+x+2)^{\frac{3}{2}} + c$$

$$\therefore \int_0^1 (6x^2+2)\sqrt{x^3+x+2}\,dx = \left.\frac{4}{3}(x^3+x+2)^{\frac{3}{2}}\right|_0^1 = \frac{4}{3}\left(2^3 - 2\sqrt{2}\right) =$$

$$= \frac{32 - 8\sqrt{2}}{3}$$

(3) $\displaystyle\int_e^3 \frac{1}{x\ln(x)}\,dx$

$$u = \ln(x), \qquad du = \frac{1}{x}dx$$

$$\int \frac{1}{x\ln(x)}\,dx = \int \frac{1}{u}\,du = \ln|u| + c = \ln|\ln(x)| + c$$

$$\therefore \int_e^3 \frac{1}{x\ln(x)}\,dx = \ln|\ln(x)|\Big|_e^3 = \ln(\ln(3)) - \ln(1) = \ln(\ln(3))$$

(4) $\displaystyle\int_0^1 \frac{e^x+1}{e^x}\,dx$

$$\int \frac{e^x+1}{e^x}\,dx = \int \left(1 + \frac{1}{e^x}\right)\,dx = \int \left(1 + e^{-x}\right)\,dx = x - e^{-x} + c$$

$$\therefore \int_0^1 \frac{e^x+1}{e^x}\,dx = \left(x - e^{-x}\right)\Big|_0^1 = 1 - \frac{1}{e} + 1 = 2 - \frac{1}{e}$$