Universidade Federal de Minas Gerais
Instituto de Ciências Exatas – ICEx
Departamento de Matemática

Cálculo Diferencial e Integral I - Teste 8 (v1)
Solução

Calcule as seguintes integrais e indique a alternativa que contém a resposta correta:

(1) $\displaystyle\int_0^1 x^2 e^{(2x+1)}\,dx$

$I = \displaystyle\int x^2 e^{(2x+1)}\,dx$

$u = x^2 \;\rightarrow\; du = 2xdx$

$dv = e^{(2x+1)}dx \;\rightarrow\; v = \frac{1}{2}e^{(2x+1)}$

$I = \dfrac{x^2}{2}e^{(2x+1)} - \displaystyle\int xe^{(2x+1)}dx$

$\bar{u} = x \;\rightarrow\; d\bar{u} = dx$

$d\bar{v} = e^{(2x+1)}dx \;\rightarrow\; \bar{v} = \frac{1}{2}e^{(2x+1)}$

$I = \dfrac{x^2}{2}e^{(2x+1)} - \dfrac{x}{2}e^{(2x+1)} + \displaystyle\int \frac{1}{2}e^{(2x+1)}dx$

$I = \dfrac{x^2}{2}e^{(2x+1)} - \dfrac{x}{2}e^{(2x+1)} + \dfrac{1}{4}e^{(2x+1)}$

$\displaystyle\int_0^1 x^2 e^{(2x+1)}\,dx = \frac{e^3}{4} - \frac{e}{4}$

(2) $\displaystyle\int_2^3 \frac{x^4+1}{x^2(x-1)}\,dx$

$$\frac{x^4+1}{x^2(x-1)} = \frac{x^4+1}{x^3-x^2} = x+1+\frac{x^2+1}{x^2(x-1)}$$
$$\frac{x^2+1}{x^2(x-1)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1} = \frac{Ax(x-1)+B(x-1)+Cx^2}{x^2(x-1)}$$
$$= \frac{(A+C)x^2+(-A+B)x-B}{x^2(x-1)}$$
$$\therefore\ B=-1,\ A=-1,\ C=2$$
$$\int_2^3 \frac{x^4+1}{x^2(x-1)}\,dx = \int_2^3 \left(x+1-\frac{1}{x}-\frac{1}{x^2}+\frac{2}{x-1}\right)dx =$$
$$= \left(\frac{x^2}{2}+x-\ln|x|+\frac{1}{x}+2\ln|x-1|\right)\Bigg|_2^3 = 10/3+3\ln(2)-\ln(3)$$

(3) $\displaystyle\int_0^1 2x\,\frac{\ln(x^2+1)}{x^2+1}\,dx$

$$u = \ln(x^2+1), \quad du = \frac{2x}{x^2+1}dx$$

$$\int u du = \frac{u^2}{2}$$

$$\int_0^1 \ln(x^2+1)\,\frac{2x}{x^2+1}\,dx = \frac{1}{2}(\ln(x^2+1))^2\Big|_0^1 = \frac{(\ln(2))^2}{2}$$

(4) $\displaystyle\int_0^\pi e^{\cos(x)}\,\text{sen}\,(x)\,dx$

$$u = \cos(x), \quad du = -\text{sen}\,(x)dx$$

$$\int -e^u du = -e^u$$

$$\int_0^\pi e^{\cos(x)}\,\text{sen}\,(x)\,dx = -e^{\cos(x)}\Big|_0^\pi = e - \frac{1}{e}$$