Universidade Federal de Minas Gerais
Instituto de Ciências Exatas – ICEx
Departamento de Matemática

Cálculo Diferencial e Integral I - Teste 8 (v1)
Solução

Calcule as seguintes integrais e indique a alternativa que contém a resposta correta:

(1) $\displaystyle\int_0^1 x^2 \cos(\pi x)\, dx$

$u = x^2, \;\to\; du = 2xdx$

$dv = \cos(\pi x)dx, \;\to\; v = \dfrac{1}{\pi}\,\mathrm{sen}\,(\pi x)$

$\displaystyle\int x^2 \cos(\pi x)\, dx = \frac{x^2}{\pi}\,\mathrm{sen}\,(\pi x) - \int \frac{2x}{\pi}\,\mathrm{sen}\,(\pi x)\, dx$

$\bar{u} = \dfrac{2x}{\pi}, \;\to\; d\bar{u} = \dfrac{2}{\pi}\, dx$

$d\bar{v} = \mathrm{sen}\,(\pi x)\, dx, \;\to\; \bar{v} = -\dfrac{1}{\pi}\cos(\pi x)$

$\displaystyle\int \frac{2x}{\pi}\,\mathrm{sen}\,(\pi x)\, dx = -\frac{2x}{\pi^2}\cos(\pi x) + \int \frac{2}{\pi^2}\cos(\pi x)dx = -\frac{2x}{\pi^2}\cos(\pi x) + \frac{2}{\pi^3}\,\mathrm{sen}\,(\pi x)$

$\displaystyle\therefore \int_0^1 x^2 \cos(\pi x)\, dx = \left(\frac{x^2}{\pi}\,\mathrm{sen}\,(\pi x) + \frac{2x}{\pi^2}\cos(\pi x) - \frac{2}{\pi^3}\,\mathrm{sen}\,(\pi x)\right)\Bigg|_0^1 = -\frac{2}{\pi^2}$

(2) $\displaystyle\int_2^3 \frac{x^3-2}{x(x-1)^2}\,dx$

$$\frac{x^3-2}{x(x-1)^2} = 1 + \frac{2x^2-x-2}{x(x-1)^2}$$
$$\frac{2x^2-x-2}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{(A+B)x^2+(-2A-B+C)x+A}{x(x-1)^2}$$
$$A=-2,\ B=4,\ C=-1$$
$$\frac{x^3-2}{x(x-1)^2} = 1 - \frac{2}{x} + \frac{4}{x-1} - \frac{1}{(x-1)^2}$$
$$\int_2^3 \frac{x^3-2}{x(x-1)^2}\,dx = \int_2^3 \left(1 - \frac{2}{x} + \frac{4}{x-1} - \frac{1}{(x-1)^2}\right)dx =$$
$$= \left(x - 2\ln|x| + 4\ln|x-1| + \frac{1}{x-1}\right)\Bigg|_2^3 = \frac{1}{2} - 2\ln(3) + 6\ln(2)$$

(3) $\displaystyle\int_1^e \frac{1}{x(1+\ln(x))}\,dx$

$$u = 1+\ln(x), \;\rightarrow\; du = \frac{1}{x}\,dx$$
$$\int \frac{1}{x(1+\ln(x))}\,dx = \int \frac{1}{u}\,du = \ln|u| + c = \ln|1+\ln(x)| + c$$
$$\int_1^e \frac{1}{x(1+\ln(x))}\,dx = \left(\ln|1+\ln(x)|\right)\Big|_1^e = \ln(2)$$

(4) $\displaystyle\int_1^e \frac{\cos(\pi\ln(x))}{x}\,dx$

$$u = \ln(x), \;\rightarrow\; du = \frac{1}{x}\,dx$$
$$\int \frac{\cos(\pi\ln(x))}{x}\,dx = \int \cos(\pi u)\,du = \frac{\operatorname{sen}(\pi u)}{\pi} + c = \frac{\operatorname{sen}(\pi\ln(x))}{\pi} + c$$
$$\int_1^e \frac{\cos(\pi\ln(x))}{x}\,dx = \left.\frac{\operatorname{sen}(\pi\ln(x))}{\pi}\right|_1^e = 0$$