Universidade Federal de Minas Gerais
Instituto de Ciências Exatas – ICEx
Departamento de Matemática

Cálculo Diferencial e Integral I - Teste 9 (v1)
Solução

Calcule as seguintes integrais e indique a alternativa que contém a resposta correta:

(1) $\displaystyle\int_0^{\frac{\pi}{2}} \cos^3 x \ \text{sen}^2 x \ dx$

$$I = \int \cos^3 x \ \text{sen}^2 x \ dx = \int \cos^2 x \ \text{sen}^2 x \ \cos x \ dx = \int (1 - \text{sen}^2 x) \ \text{sen}^2 x \ \cos x \ dx$$

$\text{sen}\, x = u \ \rightarrow \ du = \cos x \ dx$

$$I = \int (1 - u^2) u^2 \, du = \int (u^2 - u^4) \, du = \frac{u^3}{3} - \frac{u^5}{5} + c = \frac{\text{sen}^3 x}{3} - \frac{\text{sen}^5 x}{5} + c$$

$$\int_0^{\frac{\pi}{2}} \cos^3 x \ \text{sen}^2 x \ dx = \left[\frac{\text{sen}^3 x}{3} - \frac{\text{sen}^5 x}{5} \right] \Bigg|_0^{\frac{\pi}{2}} = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$$

$$\textbf{(2)} \quad \int_0^{\ln(2)} e^x \cos^3\left(\frac{\pi}{2}e^x\right)\ dx$$

$$u = e^x \ \rightarrow \ du = e^x dx$$

$$I = \int e^x \cos^3\left(\frac{\pi}{2}e^x\right)\ dx = \int \cos^3\left(\frac{\pi}{2}u\right)\ du = \int \cos^2\left(\frac{\pi}{2}u\right)\cos\left(\frac{\pi}{2}u\right)\ du =$$

$$= \int \left(1 - \operatorname{sen}^2\left(\frac{\pi}{2}u\right)\right)\cos\left(\frac{\pi}{2}u\right)\ du$$

$$w = \operatorname{sen}\left(\frac{\pi}{2}u\right) \ \rightarrow \ dw = \frac{\pi}{2}\cos\left(\frac{\pi}{2}u\right) du$$

$$I = \int \frac{2}{\pi}\left(1 - w^2\right) dw = \frac{2}{\pi}\left(w - \frac{w^3}{3}\right) + c =$$

$$= \frac{2}{\pi}\left(\operatorname{sen}\left(\frac{\pi}{2}u\right) - \frac{1}{3}\operatorname{sen}^3\left(\frac{\pi}{2}u\right)\right) + c =$$

$$= \frac{2}{\pi}\left(\operatorname{sen}\left(\frac{\pi}{2}e^x\right) - \frac{1}{3}\operatorname{sen}^3\left(\frac{\pi}{2}e^x\right)\right) + c$$

$$\int_0^{\ln(2)} e^x \cos^3\left(\frac{\pi}{2}e^x\right)\ dx = \frac{2}{\pi}\left(\operatorname{sen}\left(\frac{\pi}{2}e^x\right) - \frac{1}{3}\operatorname{sen}^3\left(\frac{\pi}{2}e^x\right)\right)\Bigg|_0^{\ln(2)} =$$

$$\frac{2}{\pi}\left[\left(\operatorname{sen}(\pi) - \frac{1}{3}\operatorname{sen}^3(\pi)\right) - \left(\operatorname{sen}\left(\frac{\pi}{2}\right) - \frac{1}{3}\operatorname{sen}^3\left(\frac{\pi}{2}\right)\right)\right] = -\frac{4}{3\pi}$$

(3) $\displaystyle\int_{-1}^{0} \frac{x+1}{x-1}\,dx$

$$\frac{x+1}{x-1} = 1 + \frac{2}{x-1}$$

$$\int_{-1}^{0} \frac{x+1}{x-1}\,dx = \int_{-1}^{0} \left(1 + \frac{2}{x-1}\right) dx =$$

$$= \left(x + 2\ln|x-1|\right)\Big|_{-1}^{0} = 2\ln(1) - (-1 + 2\ln(2)) = 1 - 2\ln(2)$$

(4) $\displaystyle\int_{1}^{2} \frac{1}{\sqrt{x-1}} dx$

$$\int_{1}^{2} \frac{1}{\sqrt{x-1}} dx = \lim_{t\to 1^+} \int_{t}^{2} \frac{1}{\sqrt{x-1}} dx = \lim_{t\to 1^+} 2\sqrt{x-1}\Big|_{t}^{2} = 2 - \lim_{t\to 1^+} 2\sqrt{t-1} = 2$$