Universidade Federal de Minas Gerais
Instituto de Ciências Exatas – ICEx
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Cálculo Diferencial e Integral I - Teste 9 (v1)
Solução

Calcule as seguintes integrais e indique a alternativa que contém a resposta correta:

(1) $\int \cos^5 x \ \text{sen}^4 x \ dx$

$I = \int \cos^5 x \ \text{sen}^4 x \ dx = \int \cos^4 x \ \text{sen}^4 x \ \cos x \ dx = \int (1 - \text{sen}^2 x)^2 \ \text{sen}^4 x \ \cos x \ dx$

$u = \text{sen}\, x \ \rightarrow \ du = \cos x \, dx$

$I = \int (1 - u^2)^2 \, u^4 \, du = \int (u^4 - 2u^6 + u^8) \, du = \dfrac{u^5}{5} - \dfrac{2u^7}{7} + \dfrac{u^9}{9} + c =$

$= \dfrac{\text{sen}^5 x}{5} - \dfrac{2\text{sen}^7 x}{7} + \dfrac{\text{sen}^9 x}{9} + c$

(2) $\displaystyle\int_0^1 x^2 \operatorname{sen}^3\left(\frac{\pi}{2}(x^3+1)\right)\ dx$

$$u = x^3+1 \ \rightarrow \ du = 3x^2dx$$

$$I = \int x^2 \operatorname{sen}^3\left(\frac{\pi}{2}(x^3+1)\right)\ dx = \frac{1}{3}\int \operatorname{sen}^3\left(\frac{\pi}{2}u\right)\ du = \frac{1}{3}\int \operatorname{sen}^2\left(\frac{\pi}{2}u\right)\ \operatorname{sen}\left(\frac{\pi}{2}u\right)\ du =$$

$$= \frac{1}{3}\int \left(1-\cos^2\left(\frac{\pi}{2}u\right)\right)\ \operatorname{sen}\left(\frac{\pi}{2}u\right)\ du$$

$$w = \cos\left(\frac{\pi}{2}u\right) \ \rightarrow \ dw = -\frac{\pi}{2}\operatorname{sen}\left(\frac{\pi}{2}u\right)du$$

$$I = \frac{-2}{3\pi}\int\left(1-w^2\right)\ dw = \frac{-2}{3\pi}\left(w-\frac{w^3}{3}\right)+c =$$

$$= \frac{-2}{3\pi}\left(\cos\left(\frac{\pi}{2}u\right)-\frac{1}{3}\cos^3\left(\frac{\pi}{2}u\right)\right)+c =$$

$$= \frac{-2}{3\pi}\left(\cos\left(\frac{\pi}{2}(x^3+1)\right)-\frac{1}{3}\cos^3\left(\frac{\pi}{2}(x^3+1)\right)\right)+c$$

$$\int_0^1 x^2 \operatorname{sen}^3\left(\frac{\pi}{2}(x^3+1)\right)\ dx = \frac{-2}{3\pi}\left(\cos\left(\frac{\pi}{2}(x^3+1)\right)-\frac{1}{3}\cos^3\left(\frac{\pi}{2}(x^3+1)\right)\right)\Bigg|_0^1 =$$

$$= \frac{-2}{3\pi}\left[\left(\cos(\pi)-\frac{1}{3}\cos^3(\pi)\right)-\left(\cos\left(\frac{\pi}{2}\right)-\frac{1}{3}\cos^3\left(\frac{\pi}{2}\right)\right)\right] = \frac{4}{9\pi}$$

(3) $\displaystyle\int_{-1}^{1} \frac{x-1}{x+1}\, dx$

$\frac{x-1}{x+1} = 1 - \frac{2}{x+1}$

$$\int_{-1}^{1} \frac{x-1}{x+1}\, dx = \int_{-1}^{1} \left(1 - \frac{2}{x+1}\right)\, dx = \lim_{t \to -1^+} \left(x - 2\ln|x+1|\right)\Big|_t^1$$

$$= 2 - 2\lim_{t \to -1^+} \left(\ln(2) - \ln(t+1)\right) = -\infty$$

(4) $\displaystyle\int_{0}^{\frac{\pi}{2}} \tan x dx$

$$I = \int \tan x dx = \int \frac{\operatorname{sen} x}{\cos x} dx$$

$$u = \cos x \;\; \to \;\; du = -\operatorname{sen} x \, dx$$

$$I = -\int \frac{1}{u} du = -\ln|u| + c = -\ln|\cos x| + c$$

$$\int_{0}^{\frac{\pi}{2}} \tan x \; dx = \lim_{t \to \frac{\pi}{2}} \left(-\ln(\cos x)\Big|_0^t\right) = \left(\lim_{t \to \frac{\pi}{2}} \left(-\ln(\cos t)\right)\right) - \ln(1) = \infty$$